

ON THE CURRENT NEED OF EXPERT RULES FOR THE TOPOLOGY OPTIMIZATION OF STRUCTURES WITH NON-LINEAR BEHAVIORS

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Abstract. *In order to optimize the topology of structures with non-linear structural behavior like crash structures, pure mathematical optimization methods do not exist. This contribution shows the possibilities of the involving of expert rules and derived heuristics in the optimization process. It is a very interesting possibility to avoid time-consuming sensitivity calculations. Two methods are evaluated more precisely: The Hybrid Cellular Automaton (HCA), which works with one special rule and the Graph and Heuristic based Topology Optimization (GHT). The opportunities to find powerful design rules generated by experts or by analysis and clustering of many simulation data (big data) are shown. Supported by benchmarks examples, this contribution shows the limitation of optimization methods without expert rules and offers an overview of the possibilities to apply expert rules as an important part in an automatic topology optimization process.*

1 PROBLEM DESCRIPTION

Topology optimization for the layout finding of structures is commonly used for linear static mechanical problems within the industry. The most often used approach is the subdividing of the topology domain in small parts (pixel or voxel) and to distinguish whether there is material or not ^[1]. E.g. the well-known homogenization method minimizes the mean compliance considering a mass constraint. These methods work very fast, because they use existing analytical sensitivities of the most relevant objectives like mean compliance, stresses or mass.

Regarding to crash-loaded structures with highly non-linear behavior, there are a lot of more complex objectives and constraints:

- Consideration of special acceleration values like the head injury criterion (HIC-value)
- Energy absorption,

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- Special force levels,
- Smooth force-displacement curve,
- Smooth acceleration-time curve,
- Special force paths for special loadcases.
- High stiffness of special parts, e.g. parts in a main force paths in the passenger area
- Low stiffness of special parts, e.g. at positions of the head contact of a pedestrian,
- Special safety criteria, e.g. no leakage of the petrol system.

In addition to these optimization functions, the behavior of the crash-loaded structures is strongly non-linear, normally calculated by the explicit finite element approach:

- Material plasticity and material failure models
- Geometric nonlinearities
- Contact phenomena
- Numerical and physical bifurcation points
- Non-smooth structural responses
- Mesh dependent results
- No analytical determination of the sensitivities (explicit time integration)
- Huge number of local optima in the design space

There are two possibilities for the definition of the design variables. The first possibility is the density of the material in the already mentioned small parts. Therefore there are millions of design variables. The second possibility is the CAD description of the structural elements e.g. by support of graph theory.

There is an urgent need of topology optimization methods for crash-loaded structures. Especially the automotive industry needs support from the optimization society, e.g. for fulfilled legal requirements. There, it is necessary to find new ideas for efficient methods. The considering of mean compliance and stress constraints is not enough. It is necessary to involve all relevant objectives and constraint functions.

2 POSSIBILITY: TOPOLOGY DERIVATIVES FOR NON-LINEAR PROBLEMS

It is very costly and often not beneficial to generate sensitivities by using explicit finite element calculations. Research activities to find efficient methods of necessary Topological Derivatives (TD) exist ^[2,3]. The idea is to find analytical or semi-analytical descriptions of the Topological Derivatives for an arbitrary state of displacements and stresses. For a functional $J(\Omega)$, the Topological Derivative is described by

$$TJ(x) = \lim_{\rho \downarrow 0} \frac{J(\Omega \setminus \overline{B_\rho(x)}) - J(\Omega)}{|B_\rho(x)|}. \quad (1)$$

Here, $B_\rho(x)$ denotes a ball (in a 3D structure) or a hole (in a 2D structure) with the radius. As an example, Figure 1 shows the scheme for the calculation of the Topology Derivatives depending on the principal stresses.

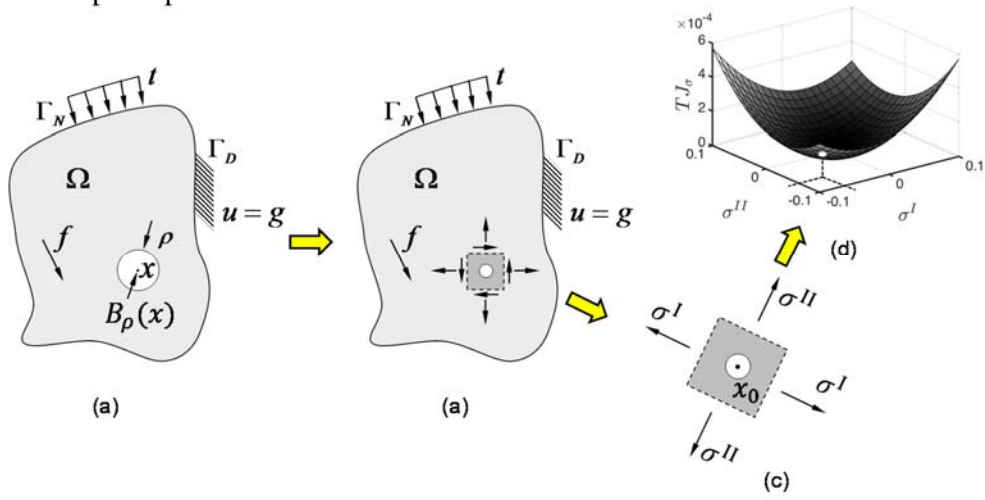


Figure 1: Calculation of Topological Derivatives depending on the principal stresses: a) mechanical Problem, b) Identification of the state in the area of the hole, c) Submodel for the numerical calculation of the Topology Derivatives, d) Meta-model of the Topological Derivatives depending on the principal stresses ^[2]

For creating the sample points for calculating the meta-model of the Topological Derivatives, the finite element model with a non-linear material behavior shown in Figure 2 is used as basic for the approximation in figure 3.

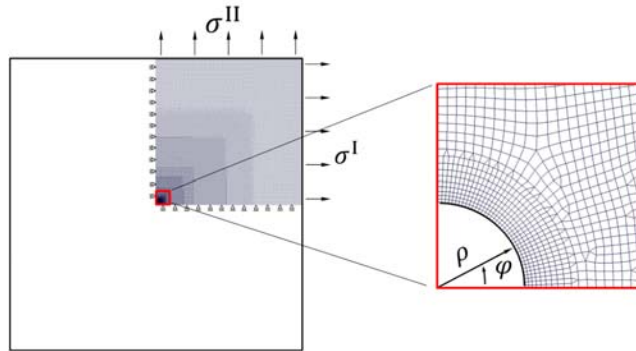


Figure 2: Finite element model with a non-linear material behavior ^[2]

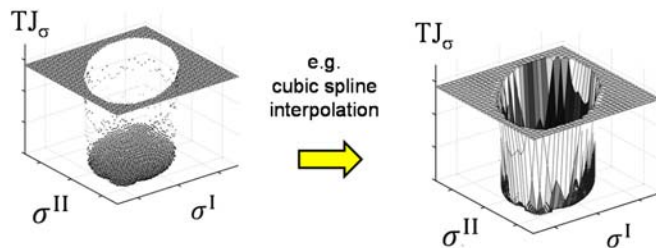


Figure 3: Approximation of the Topological Derivative depending on arbitrary principal stresses ^[2]

The research of finding Topology Derivatives is still at the beginning and it needs additional years to find a solution for structures with a highly non-linear behavior.

3 EXPERT RULES IN THE OPTIMIZATION PROCESS

Expert rules are a powerful possibility to avoid the need for the calculation of sensitivities. The idea is the addition of these automatic expert rules in the optimization process (Figure 4).

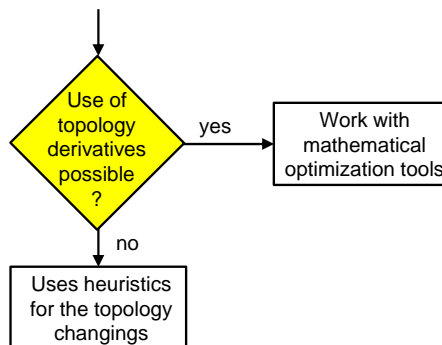


Figure 4: Practical approach for the topology optimization of structures with non-linear behavior

3.1 One-rule approaches - example: Hybrid Cellular Automaton (HCA)

The Hybrid Cellular Automaton (HCA) ^[4] is one example of an expert rule. The modification of the structure is carry out by finding a homogeneous distribution of the inner energy density. Neighboring elements in the Cellular Automaton lattice are considered. There is a direct connection of Cellular Automaton lattice and finite element mesh. The HCA is a density approach dealing with relative densities. A rule homogenizes the energy density in a way that no sensitivity calculation are necessary.

3.2 Competing rules approach - example: Graph and Heuristic based Topology Optimization (GHT)

For complex optimization tasks as mentioned in chapter 1, there is a need to consider different competing heuristics. The Graph and Heuristic based Topology Optimization (GHT) [5] combines topology, shape and sizing optimizations in one optimization process. It uses widely used finite element shell models for executing crash simulations. The optimization task is divided into an outer optimization loop, which performs the topology optimization with heuristics (derived from expert rules) and an inner optimization loop, which performs the mathematical shape optimization and sizing to evaluate the design. The heuristics use result data of finite element simulations like strains, stresses, displacement, velocities and accelerations. Based on this information the heuristics make proposals for modifications of the structure. Figure 5 shows the basic scheme of the GHT. For the flexible geometry description, mathematical graphs are used. The heuristics are used to perform structural modifications. The optimization problem is devised into two optimization loops, the outer loop for the structural modifications performed by heuristics (mainly topology changes) and the inner loop with a common shape and/or sizing optimization for a design layout, which is coming from the outer loop. The GHT has different strategies of the combination of the topology changing by heuristics and the shape optimization. E.g. the tracking of E competing designs in parallel (with $E \geq 1$) comes to a branching strategy to avoid local optima. A higher E leads to a higher probability to skip local optima, but leads to higher computer time.

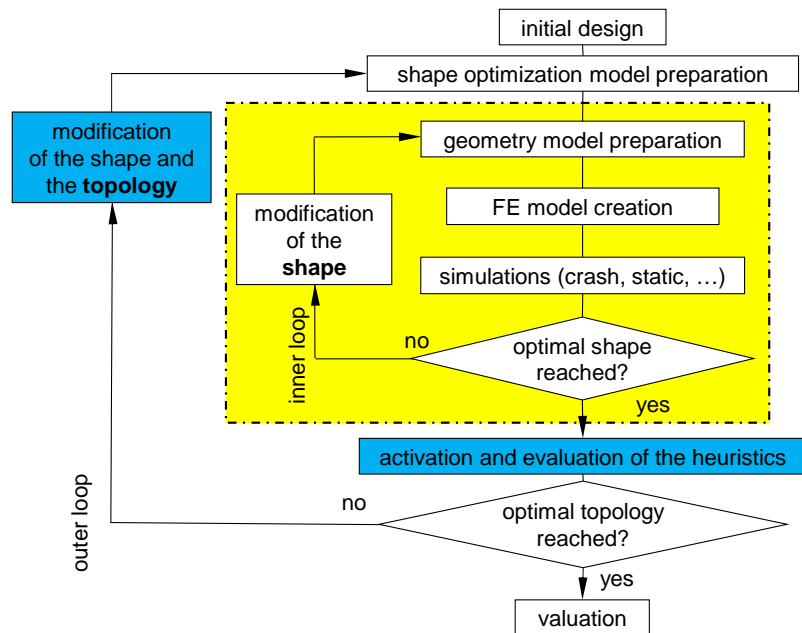


Figure 5: Basic scheme of the Graph and heuristic based topology optimization (GHT)

4 GENERATION OF RULES FROM EXPERT KNOWLEDGE

There are two possibilities for generating rules, first the organization of brainstorming meetings with experts and secondly the clustering of many simulation data (big data) ^[7]. This chapter has the focus on brainstorming meetings with crash development groups of several car producers ^[8]. Some results are sorted in the following list:

Increasing the stiffness in crash:

- Support components with buckling tendency
- Increasing of corner stiffness
- Inserting of Y-junctions
- Split high-loaded structures.
- No arch shaped components
- Use the full design domain:
- Filling of large cutouts
- If the torsion is to large, insert circular structures
- ...

Reducing the stiffness in crash:

- Including of crash elements
- Arching of straight components
- Inserting of triangle cutouts
- ...

Simplification:

- Delete unloaded components
- Use a small number of chambers
- ...

Balancing the energy density:

- Homogenize the buckling length
- Moderate changing of the wall thickness
- ...

Manufacturing constraints:

- Boundaries of the wall thicknesses,
- Boundary of the angle between two walls,
- Boundary of the distance between two walls
- ...

Based on the results of the brainstorming meetings the heuristics are implemented in the GHT software. Figure 6 shows the basic heuristics.

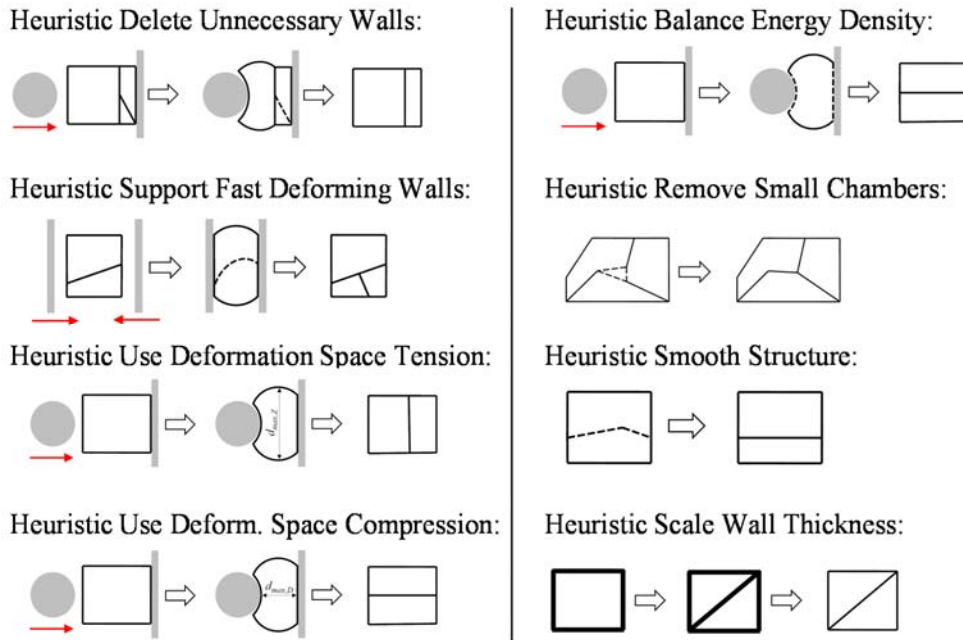


Figure 6: Examples of the implemented heuristics

5 BENCHMARK OPTIMIZATION RESULTS

Supported by benchmarks examples, this contribution shows the limitation of optimization methods without expert rules and offers an overview of the possibilities of using expert rules as an important part in an automatic topology optimization process.

5.1 Cantilever frame structure

The considered cantilever frame structure is shown in Figure 7. The optimizer has to find an optimal layout of walls in the structure. The optimization tasks are the following:

- Application 1: minimize maximum intrusion so that the frame mass ≤ 0.027 kg
- Application 2: minimize maximum acceleration so that the intrusion ≤ 49 m

The manufacturing constraints for both applications are:

- $0.5 \text{ mm} \leq \text{wall thickness} \leq 10 \text{ mm}$
- wall distance $\leq 10 \text{ mm}$
- wall connection angle $\leq 15^\circ$

The HCA can only optimize application 1 without the given manufacturing constraints. The results are shown in figure 7.

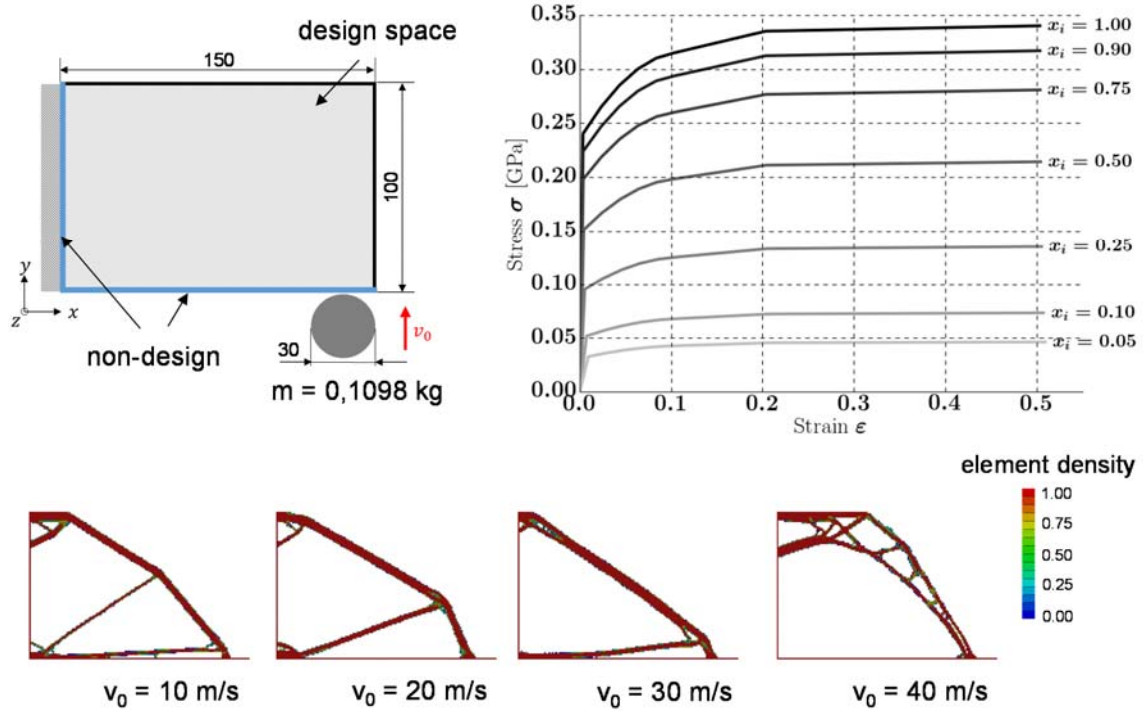


Figure 7: Maximum stiffness design with HCA for different initial velocities the mass m with the element density x_i [9]

Figure 8 shows the optimization results of the GHT.

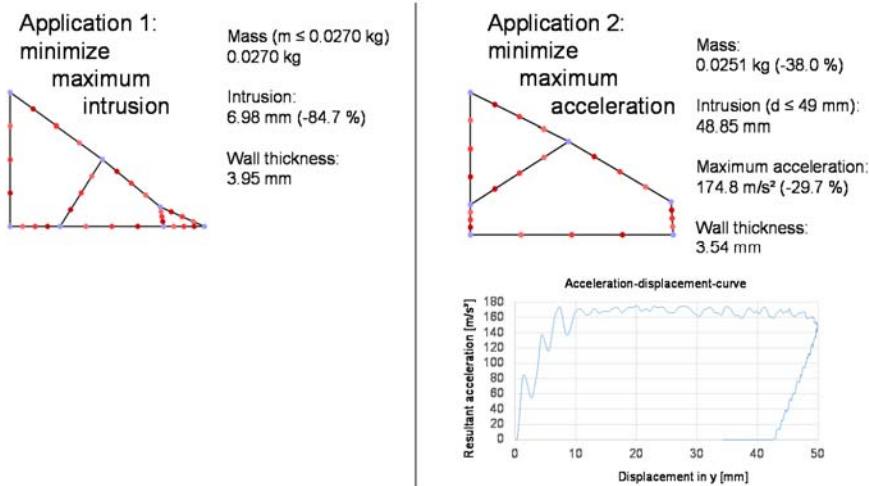


Figure 8: Maximum stiffness design with HCA for different initial velocities the mass m with the element density $v_0 = 25 \text{ m/s}$ x_i [5]

5.2 Rocker of a vehicle body-in-white structure

The optimization task is to find the optimal topology and shape of the cross section of the rocker profile shown in figure 9.

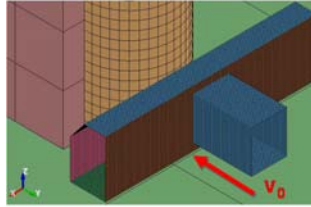


Figure 9: Mechanical problem of the rocker ^[6]

The objective is to minimize the maximal force at a moved rigid wall (velocity v_0), so that functional constraints

- mass ≤ 2.801 kg
- intrusion (pole crash) ≤ 70 mm
- stiffness(bending and torsion) ≥ 50 % stiffness initial design

and the manufacturing constraints

- $1.6 \text{ mm} \leq \text{wall thickness} \leq 3.5 \text{ mm}$
- distance of walls $\geq 10 \text{ mm}$
- connection angle of walls $\geq 15^\circ$
- maximum chamber size ration 1 : 20

are fulfilled. HCA is not able to optimize this problem, so figure 10 and 11 show only the GHT results.

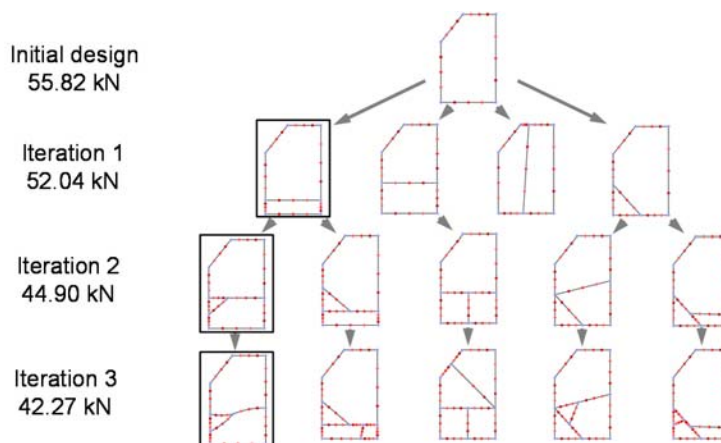


Figure 10: Optimization history (branching - competing designs) of the optimization of the rocker ^[6]

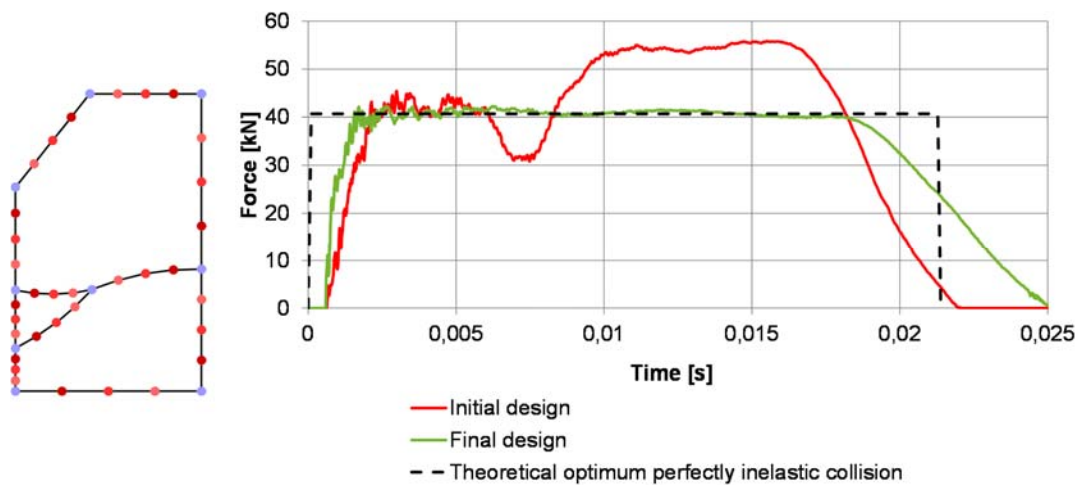


Figure 11: Optimization history (branching - competing designs) of the optimization of the rocker ^[6]

6 CONCLUSIONS

GHT and HCA as representatives of rule-based optimization methods provide interesting results, which cannot be achieved with purely mathematical methods. The expert knowledge based generation of powerful heuristics is time consuming. In the future, attention must be paid to a suitable interplay of mathematical methods and heuristics.

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